What's the cheapest way to cut ten tons?

Line up **every ton of every firm** in one row, cheapest to cut first → the cheapest way to cut N tons = the **first N steps** · test any split: **compare the last tons** · **one price** climbs the row from the bottom with nobody reading a cost sheet.

1 The chart you know

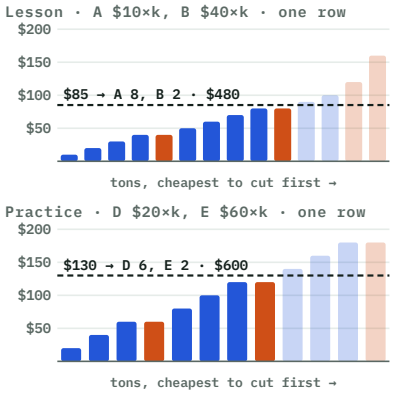
Policy 5's two factories

- Goal: **10 tons** of CO_2 (carbon dioxide, the gas that warms the planet) a day less · factory A's k-th ton costs `$10 × k` to cut, B's `$40 × k`
- **Marginal cost of cutting** = what cutting one more ton costs → rises as the easy fixes get used up
- Rule "five each": A `10 + … + 50 = $150` · B `40 + … + 200 = $600` ⇒ **$750**, four fifths at B · **your guess** ("should each cut five?"): never graded or saved

2 The island's staircase

one row, cheapest first

- Forget who owns which ton → all tons in **one row**: `10 · 20 · 30 · 40 · 40 · 50 · 60 · 70 · 80 · 80` | 90 · 100 · 120… = **marginal abatement cost curve** (abatement = cutting pollution)
- Cheapest N tons = **first N steps**, whoever cuts them → ten tons: A 8 (`$360`) + B 2 (`$120`) = **$480** vs $750 ⇒ **$270 saved, over a third** · costliest of the ten: **$80** (A's 8th and B's 2nd)
- Six tons: first six steps → A 5, B 1 = `$190` vs 3 + 3 = `$300`
- Practice · D `$20 × k`, E `$60 × k`, 8 tons: four each `200 + 600 = $800` → one row `20 · 40 · 60 · 60 · 80 · 100 · 120 · 120` ⇒ D 6, E 2 = **$600** · **a quarter** saved



3 Compare the last tons

where the saving hides

- "Five each" makes B cut tons 3-5 (`120 + 160 + 200 = $480`) and leaves A's tons 6-8 dark (`60 + 70 + 80 = $210`) → swap them: `480 − 210 = $270`, the whole saving
- **Equal marginal cost** = cheapest split ⇔ the last ton each firm cuts costs the same (**$80 = $80**) → no swap can save
- **Swap test**: one firm's last ton costlier than another's next → move it → repeat until they match · works for any split, any number of firms
- Practice · the opposite mistake, "all on D": `20 + … + 160 = $720` → swap $160 for E's $60: **−$100** → $140 for $120: **−$20** → $120 vs $180: **stop** ⇒ D 6, E 2, $600

4 One price, nobody in charge

the council reads a meter of tons cut

- Price per ton → each firm cuts every ton that costs **less than the price**, pays the price on the rest → **$85** ⇒ A 8, B 2 (any price $80-90) · practice **$130** ⇒ D 6, E 2 (any $120-140)
- New factory C `$20 × k` at $85 → cuts 4 ⇒ meter **14** → council lowers the price → **$65**: A 6, B 1, C 3 = 10 tons, `$370` · practice: F `$40 × k`, $130 → meter 11 → $100-120: D 5, E 1, F 2 = 8 tons, `$480`
- **Groping (tâtonnement)** = prices found by trial: call a price, read the offer, call again · **Léon Walras** (French economist, 1834-1910; the name of this site; Thinkers 4)
- **Price-based tool** = tax · payment per ton · permits that trade → one number per ton ⇒ cheap tons first | **fixes the cuts**: five each · same filter for all · permits that cannot trade

5 Sulfur, 1990

and B's town

- **EPA** (US Environmental Protection Agency) · Clean Air Act Amendments **1990**: SO_2 (sulfur dioxide → acid rain) **10 million tons** a year below 1980 · from **1995** (biggest plants; all from 2000) one **allowance** (= permit) per ton, unused ones sold → cap (the most tons allowed) fixes the total, trading picks **where**
- Acid in eastern US rain (sulfate = the sulfur it carries) 1989-91 → 2020-22: **down more than 70%** (EPA) · costs "far lower than expected" · ≈ **$186 a ton** in 1995 (study at MIT, US university, 1997; Ellerman, Schmalensee et al., *Markets for Clean Air*, 2000)
- Not proof alone: scrubbers (sulfur filters) cheaper than forecast · the 95% fall since 1990 counts later rules too · says nothing on whether 10 million tons was the right goal (Lessons 1, 4)
- **Hot spot** = a place with far more of a **local** pollutant (soot = black dust of burning, sulfur) because the cheap cuts happened elsewhere · CO_2 mixes everywhere → no hot spot · **for** the cheapest split: pay B's town from the $270 · **against**: a town may refuse eight tons whatever the saving · the council's choice, not the lesson's
- **Ten questions to ask about any green policy** (so far): **1** · What does the smoke really cost, and who pays that cost now? · **2** · Who owns the resource? What nobody owns, everybody overuses. · **3** · **Does it cut where cutting is cheapest?**

Watch out: "five each" may be **fair**; it is **not cheap** unless the staircases match · "the cheap firm does it all" is wrong too · cheapest split ⇔ **last tons equal**, not equal tons · a trading cap still fixes the **total**