Should you confess?

Two players, two moves each, one table. Find each best reply; if one move is best whatever the other does, it is dominant. When both play it and both end worse off: a prisoner's dilemma.

1 The game

the deal as a 2×2 table

- **Game theory**: choices where what you get depends on others' choices too · each chooser = a **player** · **payoff** = what a player gets in a cell (here: years in prison)
- Ana + Ben arrested · separate rooms · no talking · same deal for each · **your move** first (confess or silent: never graded, never stored)
- Both silent → **1 yr** each · one confesses → the confessor goes **free**, the other **10 yrs** · both confess → **5 yrs** each
- **Payoff table**: rows = Ana's moves (blue) · columns = Ben's (orange) · cell = `Ana / Ben`, Ana first · years ⇒ **fewer is better** · profit ⇒ more is better

2 Best reply

→ dominant strategy

- **Best reply**: "the move that gives a player the best result, given what the other player does"
- Method → fix Ben's move = one **column** · compare Ana's 2 blue numbers · **circle** the better · Ben: **row** by row, orange numbers
- Ben silent: confess 0 < silent 1 · Ben confesses: confess 5 < silent 10 ⇒ confess both times
- **Dominant strategy**: "a move that is the best reply to every move the other player can make" · confess = Ana's and Ben's

3 The dilemma

smart alone, stuck together

- Cell with **2 circles** = where they end = **equilibrium** → nobody gains by switching alone · here **5 / 5** (full name → Lesson 2)
- Yet **1 / 1** (both silent) is better for both · 0 / 10 better for Ana only
- **Prisoner's dilemma**: "a game where each player's best reply leaves both worse off than another outcome they could have reached together"
- 2 tests: ① each has a dominant strategy ② both using it ⇒ worse off than another cell

LESSON · YEARS

	Ben Silent	Ben Confess
Ana Silent	1 / 1	10 / (0)
Ana Confess	(0) / 10	(5) / (5)

PRACTICE · NEW DEAL

	Ben Silent	Ben Confess
Ana Silent	2 / 2	8 / (0)
Ana Confess	(0) / 8	(4) / (4)

○ = best reply ▢ = where they end ⬚ = better for both

4 Where it shows up

1950 → today

- **1950** · Merrill Flood + Melvin Dresher (US mathematicians) test the game at **RAND** (California research group, strategy for the US military) · Albert Tucker (Princeton mathematician) → the 2-prisoner story + name
- Same shape: nuclear **arms race** (US vs Soviet Union, 1950s) · gas stations, Micro 8: both high price $6,000 each → both low $4,000 each · doping · group project · price wars · taxes on trade
- Dilemma ✓ pizzerias, sprinters, airlines, two countries, shared garden · × same movie / same café (copy the other: no dominant strategy) · × open late (best reply depends) · × fences (dominant, but best for both: no trap)

5 A way out

change the payoffs

- Gang punishes talking: **+6 yrs** wherever a player confesses → 1/1 · 10/6 · 6/10 · 11/11 ⇒ silent dominant (1 < 6, 10 < 11) ⇒ end at **1 / 1** · nobody kinder: payoffs changed
- How big? > what confessing saves against **each** move · practice: saves 2 – 0 = **2** and 8 – 4 = **4** ⇒ > 4 ⇒ **5 yrs** (4 = a tie, 8 vs 8) ⇒ 2/2 · 8/5 · 5/8 · 9/9 ⇒ end at 2 / 2
- Arms race: fear of the other → building = best reply either way → both build → both spend more, neither safer · way out = treaty with **checks + penalties** (a promise alone changes no number)
- Other exits: contracts, penalties, meeting again and again → Lessons 3 and 5

6 One dominant move

practice: two shops

- Profit in $100s a night, **more is better** · Ana Late: 4 > 3, 6 > 5 ⇒ **dominant** · Ben: Early if Ana late (3 > 1), Late if Ana early (5 > 4) ⇒ none
- Ben predicts Ana's dominant move (Late) → his best reply = Early ⇒ end at **6 / 3** · no cell better for both ⇒ **not** a dilemma

TWO SHOPS · $100S

	Ben Late	Ben Early
Ana Late	(4) / 1	(6) / (3)
Ana Early	3 / (5)	5 / 4

Watch out: check "fewer" or "more is better" first · Ana's number is the first · a tie ≠ a reason to switch · a dominant strategy alone ≠ a dilemma (fences) · "best for me" ≠ "best for both"