Can an island save its way to riches?

The Solow model per worker: machines saved against machines worn out, the steady state where they cancel, why saving more raises the level of income but not its growth, and why only new ideas keep income growing.

1 Per worker

k → y

- **Capital per worker (k)** = machines, ovens, boats each worker has · **output per worker (y)** = what a worker makes in a year, in **$10,000**
- **Diminishing returns** (Micro 5): each extra machine adds less · y = 1, 2, 3, 4, 5 needs 1, 3, 5, 7, 9 more machines · k 16 → 25 ⇒ y only +1
- Two islands (Macro 10): 1% a year ⇒ doubles in 70 years · 3% ⇒ ≈ **23** years · `years to double ≈ 70 ÷ growth rate`

K	0	1	4	9	16	25	36
Y	**0**	**1**	**2**	**3**	**4**	**5**	**6**

2 Saved vs worn out

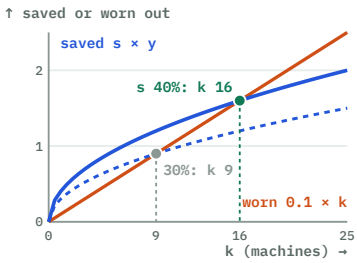
the change in k

- **Saving** = **investment** (Lesson 1) · saving rate **s = 30%** ⇒ new machines `s × y`
- **Depreciation** = machines that wear out each year: `0.1 × k` (10%)
- `change in k = s × y − 0.1 × k` · k 1: 0.3 − 0.1 = **+0.2** · k 4: 0.6 − 0.4 = **+0.2** · k 9: 0.9 − 0.9 = **0** · k 16: 1.2 − 1.6 = **−0.4**
- Palmera saves 20% ⇒ k 9: 0.6 − 0.9 = −0.3 · stops at k 4, y 2 = **$20,000**

3 Steady state

where k stops

- **Steady state** = k where saving just replaces wear-out: `s × y = 0.1 × k` · Coralia: 0.3 × 3 = 0.1 × 9 ⇒ **k = 9**, y = 3 = **$30,000**
- Below it ⇒ k grows · above it ⇒ k shrinks · a storm or a gift moves only k ⇒ back to the stop (s 50%: storm 25 → 16, 2.0 > 1.6 ⇒ back to 25)
- Robert **Solow** (American) and Trevor **Swan** (Australian), **1956**, independently · Solow: Nobel **1987**
- What moves what: s or the recipe ⇒ **saving curve** · wear-out rate ⇒ **the line** · storm, gift ⇒ **only the dot**



4 Level effect

save more?

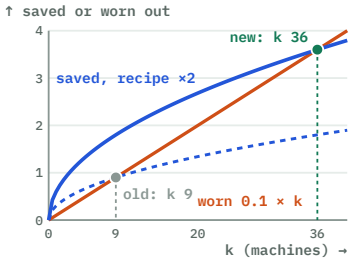
- s 30% → **40%** ⇒ stop moves to **k = 16**, y 4 = **$40,000** · year 10: k 11.6, $34,000 · year 30: k 14.3, $38,000 ⇒ fast at first, then **stops**
- **Level effect** = income moves to a new level, then stops rising ≠ growth effect · s 50% ⇒ k 25, **$50,000**
- **Golden rule** = saving rate with the most **consumption** (y − saved) in the steady state · Edmund Phelps, American, **1961** · model does not pick it for a country

S	30%	40%	50%	60%
Y · SAVED	3 · 0.9	4 · 1.6	5 · 2.5	6 · 3.6
CONSUMED	**2.1**	**2.4**	**2.5**	**2.4**

5 Ideas & catch-up

what lasts

- **Technology** = knowledge that lets the same machines make more ⇒ lifts the **whole table** · k 9: y 3 → 4.5 (s, k, wear-out unchanged) · doubled recipe: saved 1.8 > 0.9 ⇒ new stop **k 36**, y 12
- Solow **1957**: ≈ **7/8** of the rise in US output per hour, 1909-1949, not more capital · Paul Romer, American, **1990**: firms spend to find ideas
- **Convergence** = little capital ⇒ faster growth ⇒ gap narrows · GDP per person 1950 → 1973 (Maddison Project 2020, past incomes): Japan ×6, **8.1%** a year · West Germany ×3, 5.0% · US ×1.75, 2.4% · not guaranteed: needs institutions (Development 2)
- k 1 vs k 4, both +0.2 a year ⇒ +20% vs +5% · Mankiw, David Romer (not Paul) and Weil, **1992**: add skills (human capital)



6 Watch out

the classic mistakes

- Watch out: “save more ⇒ grow faster **for ever**” × ⇒ saving raises the **level**, growth fades · growth ≠ “just more machines” ⇒ most of it comes from **ideas**
- Next at university: the Solow model with **human capital** · growth models where ideas come from **research**

Eleven ideas behind the big picture · 1 Spending decides output · 2 A lower interest rate means more spending: the IS curve · 3 The Bank picks the rate; the IS curve then picks the output · 4 AD is IS–LM at every price level · 5 Full employment is where wage setters and price setters agree · 6 Inflation is expected inflation plus the gap: you cannot fool everyone twice · 7 The Bank moves the real rate, and at zero it runs out of room · 8 Debt grows with the interest rate and shrinks with growth · **9 Saving raises the level of income; only ideas raise its growth**